Segmentation Approach Based on Topological Derivative and Level Set

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Abstract—In this paper we propose a segmentation approach that applies the topological derivative as a pre-processing step. The obtained result is used for initializing a level set model in order to get the final result. First, the method uses a low-pass filter and the topological derivative to get a rough definition of the boundaries of interest. Then, morphological operators are applied to fill holes and discard artifacts. Finally, a level set model is used to improve the result giving the desired approximation. We test the pipeline for cell image segmentation.

Keywords-image analysis;boundary extraction;topological derivative, level set.

I. INTRODUCTION

Segmentation is a fundamental step for medical imaging analysis and computer vision tasks. Approaches in image segmentation can be roughly classified in [1]: (a)Contour Based methods, like snakes and active shape models; (b)Region based techniques; (c)Optimization approaches; (d)Clustering methods, like k-means, Fuzzy C-means, Hierarchical clustering; (e)Thresholding methods. In this paper we focus on contour based techniques, the level set approach, and an optimization method formulated through the topological derivative [2].

The level set model has been successfully applied for boundary extraction, motion tracking and segmentation, mainly in medical imaging [3]. In general, the process of boundary extraction in such applications involves some kind of pre-processing step [1]. On the other hand, segmentation techniques based on topological derivative needs (in general) a post-processing step, in order to improve the results [2].

In this paper we focus on image processing through topological derivative concept [4,5,6], which quantifies the sensitivity of a given shape functional with respect to a singular domain perturbation, such as the nucleation of holes, inclusions, source-terms or even cracks. This concept, initially conceived to deal with topology optimization problems, has also been successfully applied to image segmentation [2]. Despite the observed potential of the topological derivative, its result must be improved by a contour based approach, like level set. In this way, we are combining two segmentation methods: an optimization technique to get a first approximation of the boundary and the level set method to complete the segmentation. Besides, some low-pass filters and morphological operators can be also applied in order to improve the segmentation efficiency. The obtained segmentation pipeline is the contribution of this paper: (a)Gaussian Filter; (b)Topological Derivative; (c)Mathematical Morphology; (d)Level Set.

Despite of some theoretical study about connections between level set and the topological derivative [7], the combination of these techniques has not been deeper explored in the image segmentation literature yet. One advantage of using level set in the last step is the possibility of exploring the topological capabilities of level set for multi-object segmentation and the fact that the methodology remains the same for 2D and 3D images. However, we can replace the level set for any other suitable deformable model.

II. LEVEL SET

The main idea of the level set method is to represent the deformable surface (or curve) as a level set $\{x \in \Re^3 \mid G(x) = 0\}$ of an embedding function:

$$G : \Re^3 \times \Re^+ \rightarrow \Re, \tag{1}$$

such that the deformable surface (also called front in this formulation), at time t, is given by:

$$S(t) = \{x \in \Re^3 \mid G(x,t) = 0\} \tag{2}$$

In this paper, the governing equation for the embedding function G, and, consequently, for the zero level set $S(t)$, has the general form [3]:

$$G_t = \left[\left(\frac{1+\lambda k}{1+|\nabla I|^2}\right)|\nabla G| - \gamma(\nabla|\nabla I| \bullet \nabla G)\right], \tag{3}$$

where λ and γ are parameters that weight the different terms, k is the fronts curvature, G is an embedding function, I is the image field and ∇ is the gradient operator. An initial condition $G(x, t=0)$ is also required which can be obtained through a signed-distance function as follows:

$$G(x, t=0) = \pm d, \tag{4}$$

where d is the distance from x to the surface $S(x,t=0)$ and the sign indicates if the point is interior (-) or exterior (+) to the initial front.

Finite difference schemes, based on an uniform grid [8], can be used to solve (3). Besides, the update of the embedding function can be made cheaper if the narrow-band technique is applied [3]. Also, a stopping criterion is adopted based on small displacements of the zero level set.

III. TOPOLOGICAL DERIVATIVE IN IMAGE SEGMENTATION

Let Ψ be a given shape function and consider that we introduce a singular perturbation in the original domain governed by a small parameter ε . Then, the concept of topological asymptotic expansion can be stated as:

$$\Psi(\varepsilon)=\Psi(0)+f(\varepsilon)D_T\Psi+\mathrm{O}(f(\varepsilon)) \tag{5}$$

where $\Psi(\varepsilon)$ is the value of the shape function in the perturbed domain, $\Psi(0)$ is the same shape function in the original domain and $f(\varepsilon)$ goes to zero with ε. Therefore, $D_T\Psi$ is the topological derivative of the shape function Ψ. Then, (5) provides a first order approximation of $\Psi(\varepsilon)$ for a sufficiently small ε.

More precisely, the topological derivative $D_T\Psi$ is a scalar function defined over the original domain that indicates, in each point, the sensitivity of the shape function when a singular perturbation of size ε is introduced at that point. In general, the domain singular perturbation can be, for instance: the introduction of holes, cracks or non smooth changes in the parameters of the problem (e.g., material properties, sources acting over the domain, boundary conditions, etc.).

In [2] was proposed a shape function that quantifies the misfit between the input image ν being segmented and a possible segmentation u. Let us first define the input image ν as

$$\nu \in V=\{w\in L^2(\Omega): w \text{ is constant at each element}\} \tag{6}$$

and the segmented image u as

$$u\in U=\{u\in V: u(x)\in C, \forall x\in \Omega\} \tag{7}$$

where Ω is an open bounded domain in $\Re^n, n=2,3$ and the set of classes C is given by

$$C=\{c_i\in\Re : i=1,...,N_c\}, \tag{8}$$

with N_c used to denote the number of classes in which the original image ν will be segmented and c_i represents the intensity that characterizes the i_{th} - class. Let us introduce the following shape function [2].

$$\Psi(0):=J(\varphi)=\frac{1}{2}\int_\Omega K\nabla\varphi\bullet\nabla\varphi d\Omega+ \\ +\frac{1}{2}\int_\Omega(\varphi-(\nu-u)^2 d\Omega, \tag{9}$$

where the field φ accounts for the misfit between ν and u, and is solution of the following variational problem: for all $\eta\in H^1(\Omega)$, find $\varphi\in H^1(\Omega)$, such that:

$$\int_\Omega K\nabla\varphi\bullet\nabla\eta d\Omega+\int_\Omega \varphi\eta d\Omega-\beta\int_\Omega(\nu-u)\eta d\Omega=0, \tag{10}$$

The diffusivity second order tensor field K is constant at image element level and $0<\beta\le 1$ is used to adjust the numerical algorithm. Note that the image u and function φ can be seen as the control and the state, respectively. Therefore, the image segmentation problem can be stated as following: given the image data $\nu\in V$ find the segmented image $u^*\in U$ such that minimizes a functional $J:U\rightarrow\Re$.

Associated to φ we define the function φ_ε solution to the perturbed variational problem. In this context, the perturbation is characterized by changing the segmented image u for a new one u_T that is identical to u everywhere in Ω except in a small region ω_ε centered in point $\hat{x}\in\Omega$. In ω_ε, u_T assumes one of the values $c_i\in C$. More precisely, $u_T(x)=u(x)$ if $x\in\Omega\backslash\varpi_\varepsilon$ and $u_T(x)=c_i$ if $x\in\omega_\varepsilon$. In this way, the perturbed shape function becomes

$$\Psi(\varepsilon):=J_\varepsilon(\varphi_\varepsilon)=\frac{1}{2}\int_\Omega K\nabla\varphi_\varepsilon\bullet\nabla\varphi_\varepsilon d\Omega+ \\ +\frac{1}{2}\int_\Omega(\varphi_\varepsilon-(\nu-u_T))^2 d\Omega, \tag{11}$$

where the field φ_ε is the solution of the perturbed variational problem: for all $\eta\in H^1(\Omega)$, find $\varphi_\varepsilon\in H^1(\Omega)$ such that

$$\int_\Omega K\nabla\varphi_\varepsilon\bullet\nabla\eta d\Omega+\int_\Omega\varphi_\varepsilon\eta d\Omega=\beta\int_\Omega(\nu-u_T)\eta d\Omega \tag{12}$$

The associated topological derivative can be easily calculated, namely (see [2] for details),

$$D_T\Psi(\hat{x},c_i)=\frac{1}{2}(c_i-u(\hat{x}))[(\varphi(\hat{x})-(\nu(\hat{x})-u(\hat{x})+ \\ +(\varphi(\hat{x})-(\nu(\hat{x})-c_i))+2(1-\beta)\varphi(\hat{x})], \tag{13}$$

with $f(\varepsilon)=|\omega_\varepsilon|$, $\forall\hat{x}\in\Omega$. This derivative allows us to select, for each point $\hat{x}\in\Omega$, the class $c_i\in C$ that produces the minimal value of the shape function.

It is important to emphasize that the diffusivity tensor K in general can be adopted as a homogeneous isotropic tensor kI , where k is a piecewise constant function and I is the identity tensor. Nevertheless, when the noise reduction is performed using a non-linear anisotropic method or a restoration method based on the topological derivative, the tensor K might be chosen equal to the diffusivity tensor provided by these methods.

IV. PROPOSED METHOD

The proposed segmentation methodology is composed by the following pipeline: (a)Gaussian Filter; (b)Topological Derivative; (c)Mathematical Morphology operators; (d) Level Set. The steps (a) and (c) are useful to improve the robustness of the pipeline against parameters choice.

Medical images have a complex intensity field and texture patterns. Thus, we apply low-pass filter to smooth the original image before using the topological derivative. As mentioned in section III, for the image $v \in V$ we need to find the segmented image $u^* \in U$ that minimizes the shape function J (expression (9)) by successively selecting for each point $\hat{x} \in \Omega$ the class that produces a negative value of the topological derivative. Table I describes the whole algorithm.

TABLE I. ALGORITHM I: IMAGE SEGMENTATION BASED ON THE TOPOLOGICAL DERIVATIVE

	Input: An image $v \in V$, the initial set C, an initial guess $u \in U$, the diffusivity tensor field K and the parameters β and $\alpha \in (0,1)$.
1.	**Output:** The segmented image $u^* \in U$.
2	**while** $D_T^* < 0$ **do**
3	find the solution φ for the problem (10)
4	evaluate $D_T\Psi(\hat{x}, c_i)$ according to (13)
5	compute $D_T^* = \min_{\hat{x}\in\Omega}\{D_T\Psi(\hat{x}, c_i), i = 1,\ldots,N_c\}$
6	**for** each pixel $\hat{x} \in \Omega$ **do**
7	compute $c^*(\hat{x}) = \arg\min_{c_i \in C}\{D_T\Psi(\hat{x}), c_i\}$
8	**if** $D_T\Psi(\hat{x}, c^*) \le (1-\alpha)D_T^*$, then $u(\hat{x}) = c^*$
9	**end for**
10	Update the class C according to algorithm in Table II
11	**end while**
12	$u^* = u$

Obviously, a fundamental question is how to define the set of classes C. This is performed by calling the Algorithm II, described in Table II after each interaction of the main loop (line 10 of Table I). Basically, the Algorithm II takes an initial guess C and computes C^l, $l = -1, 0, 1$, and replaces the class C by the C^l that minimizes the functional J in expression (9). The obtained result may have holes inside the objects of interest as well as artifacts in the background. These problems can be easily removed through simple morphological operators (erosion, dilation, region filling). The obtained result is binarized (0 for the background and 1 inside the objects) which can be used to get a rough approximation of the boundaries of the targets.

TABLE II. ALGORITHM II: ADJUST THE VALUES OF THE CLASSES

Input: An image $v \in V$, the set C, and the segmented image at iteration I, $u \in U$.
Output: The new set of classes C^*.
$C^* = []$
for $c_i \in C$ **do**
for $l = -1$ to 1 **do**
set $C^l = (C - \{c_i\}) \cup (c_i + l\}$
set u^l substituting $C \to C^l$
compute $j^l = J(\varphi)$
end for
$c^{\min} = c_i + l$ where $l = \min_l(j^l)$
$C^* = C^* \cup c^{\min}$
end for

The obtained curves are the input for the computation of the signed-distance function of expression (4) to initialize the level set method, described in section II. In this way, we get a first approximation of the boundary which can save time computation and improve the accuracy of the level set result.

V. EXPERIMENTAL RESULTS

In this section we show the robustness and efficiency of the proposed segmentation approach. The case study is cell segmentation. Therefore, it is assumed that we may have more than one object of interest in the image. However, we are supposing that each object boundary have the properties of connectedness and closedness. Therefore, we can fill inner holes and we can discard foreground regions linked with the image boundary. In this case we assume two classes. The initial guess for the set C is the minimum and maximum intensities of the original image at startup. The setting of parameters was performed through experimentation.

As an example, let us observe Fig.1 whose resolution is 144×150, which shows a result obtained with our method. The Fig.1(b) draws the topological derivative result which must be improved by discarding cell regions linked with the image boundary. The result is suitable for level set initialization (Fig.1(c)). The desired result, shown in Fig.1(d), is obtained after 9 interactions of the level set. The values of the parameters used are: for the topological derivative we set $\beta = 0.3$, $\alpha = 1$ and $K = 20$ and for level set we use $niter = 100$, $\Delta t = 0.005$, $e = 0.0001$, $\lambda = 13$ and $\gamma = 10$. We observe that the boundary of the cell (Fig.1(d)) in the right-hand corner of the bottom becomes smoother.

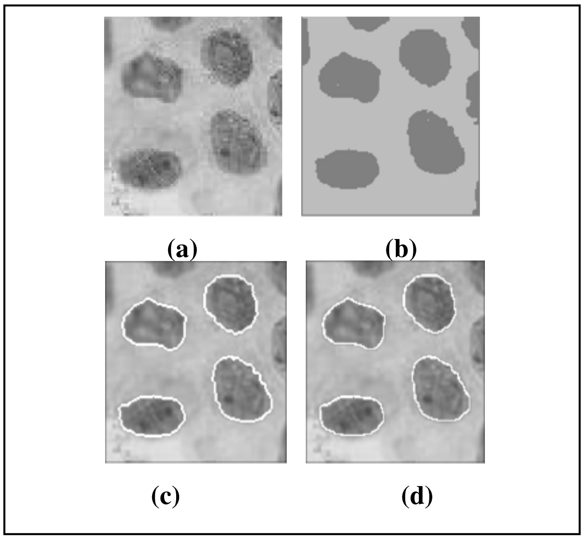

Figure 1. (a) Cell image. (b) Result of the topological. (c) Level set initialization. (d) Level set result after 9 interactions.

Another point is the scale of the objects of interest. In general, we can incorporate information about the scale range (size) of the objects and to use this information to discard artifacts that may appear in the field generated by the topological derivative. The next example, pictured on Fig.2, shows an electronic micrography of a nucleous whose resolution is 137×179. We can observe the presence of noise and artifacts. Besides, the interested boundary has points with high curvature.

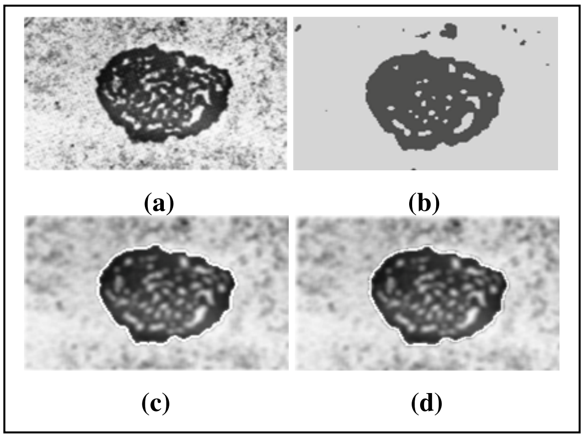

Figure 2. (a) Nucleous image. (b) Topological derivative result with inner holes and artifacts. (c) Level set initialization after application of morphological operators. (d) Level set result after 20 interactions.

The Fig.2(b) shows the result of the topological derivative. We observe small holes inside the region of interest as well as artifacts in the background due to inhomogeneities in the intensity pattern. So, we apply morphological operators in order to fill holes and a scale threshold to discard foreground artifacts. The result allows to get a suitable initialization for the level set method, as we can see in Fig.2(c). After 20 interactions we obtain the desired result. We only change the parameter e which in this case is $e = 0.00001$.

A visual inspection shows little changes in the previous examples. However, if we add noise to the original image, the behavior of step (b) of the algorithm changes, which justifies the inclusion of step (c) in the proposed pipeline. The Fig.3(a) shows the original image in Fig.2(a) after adding a gaussian noise of mean null and variance 0.8. In this case, the initialization of the level set (also shown on Fig.3(a)) is not so close to the desired boundary as we observed in the above examples. Despite of this, a visual inspections shows that we get the desired result after 135 interactions. The values of the parameters used are: $\beta = 0.2$, $niter = 400$, $e = 0.00001$ and $\lambda = 30$. The others are the same of example 1.

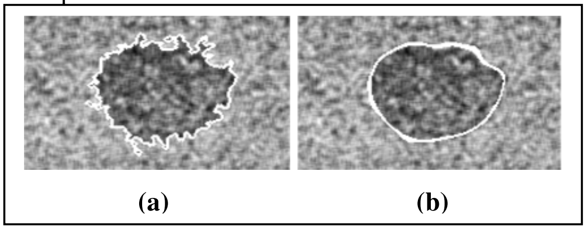

Figure 3. (a) Level set initialization after application of morphological operators. (b) Level set result after 135 interactions.

VI. CONCLUSIONS AND FUTURE WORKS

In this work we propose a segmentation approach based on topological derivative and level set method. The method applies also prior knowledge about the scale of the objects of interest. The result shows that the technique is robust against noise and very powerful for multi-object segmentation. A further direction is to study the sensitivity of the pipeline against parameter choice and to compare the performance with other related techniques.

ACKNOWLEDGMENT

We would like to acknowledge the PCI-LNCC for the financial support for this work.

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