# Using 3D-Snake for Measuring the Length of Artificially Generated Electric Arcs 

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#### Abstract

In this paper a new approach is proposed for the use of 3D-snakes in the evaluation of variations in the length of artificially generated electrical arcs. This active contour is geometrically represented by a B-spline and evolves in 3D space constrained by internal and external forces, it happens in an energy minimization procedure ("internal energy" and "external energy" is used in a sense related to visual images processing and not to physical energy related to the electrical arc behavior). This approach presents some important aspects in comparison with the ad hoc strategies found in the literature for recovering 3D geometry of electrical discharges. In addition, the proposal discussed in this paper is capable of tracking the evolution of the electrical discharge taking into account the time dependence between consecutive pairs of images.


Keywords- Electric Arc; Computer Vision; Image Processing.

## I. INTRODUCTION

The major part of faults in Brazilian electrical system are single-phase and transitory what makes possible to apply the electrotechnical maneuver called Single-Phase Auto-Reclosure (SPAR) switching to normalize the system. Provided that such maneuver consists in opening just the faulty phase during a short time, minimizing the disturbance in the electrical system, the National Electrical Energy Regulatory Agency (ANEEL) has been recommended that new transmission lines must be capable to carry out such switching, since then a mathematical model of electric arcs has been pursued and the elongation of the length of the electrical arc considered an important parameter for this model [1], [2], [3]. The research in this field has been carried out and /or supported mainly by COPPE/UFRJ, FURNAS (Brazilian utility) ANEEL 2001/2002, UNICAMP (University of Campinas), CEPEL (Brazilian High Power Laboratory) and, more recently, UDESC (Santa Catarina State University).

With no intent to exclude the purely electrotechnical strategies, the main contribution of this work is the novel application of a Computer Vision approach based on 3Dsnake which models the length of the medial axis of an electrical arc in spatial evolution. The electrical discharges are artificially generated in open field in actual electrical transmission towers at CEPEL and the 3D-snake is used for tracking and measuring the arc, providing the length in demand.

MacAlpine et al. [4], [5], [6], [7] and [8] present some important results in applying Computer Vision techniques in images of electrical discharges, nevertheless their strategies are strongly dependent on explicit methods of homologous determination, besides, the methods are applied for electrical discharges with low level of waviness dealing with not so long arcs obtained in indoor-test sites (typical ad-hoc workbench experiments). In this paper it is proposed a new approach, based on 3D-snakes for modeling the longitudinal three-dimensional medial-axes of electrical arcs, which minimizes the necessity of correspondence determination and, differently from the works cited above, there are practically no restrictions on positioning of cameras. This aspect is not presented in the strategies cited above and turns the 3D-snake model more adaptable to the out-doors environment at CEPEL.

The 3D-snake is a model based on the same principles of the precursory two-dimensional snake [9] the difference is that 3D-snake extracts its external energy from more than one image, so that the external force must be composed in 3D from a stereo pair of images. In [10] it is already described initial experiments in which 3D-snakes are applied as a model of the axis of short electrical discharges recorded by a pair of cameras.

The remainder of this paper is organized as follows. In the next Section (II) the 3D-snake model is discussed, in Section (III) a description of the experiment development and the approach applied on images of electrical discharges are given. Since the 3D-snake approach demands the knowledge of the projection matrices of the cameras, in Section (IV) is described the calibration procedure which provides these data. The results of the experiments are reported in Section (V) and the conclusions in Section (VI).

## II. 3D-SNAKE

The 2D precursory model described in [9] is similar to the 3D-snake except that, in the latter model, the functional has external energies defined in 3D space and the external forces act upon the control points of a Bspline. The optimization of the functional constrains the 3D-snake to evolve itself in 3D space in order to match its projections to the pair of features of interest in two images. The stereo pairs of images are the primary sources of external forces, each one is processed generating the respective pair of structures named (in
this work) as vector maps, these are the final sources of external forces.

## A. The functional of energy

Given (1), where Q represents a point generated by the B -spline through the set of control points V and the matrix of basis functions N , the adjustment of the 3Dsnake is defined by the functional shown in (2), where $\mathrm{E}(\mathrm{Q})$ defines the total energy of the system, $\mathrm{E}_{\text {int }}$ represents the internal energy component which preserves smoothness, at the same time that $\mathrm{E}_{\text {ext }}$ (external energy component) is responsible for the forces that attract the 3D-snake, pushing it into the features of interest captured in the images. This balance of energies must be optimized in order to the system gets a minimum of total energy and this procedure is done by the relaxation shown in (3) under constraints of internal and external forces [11].

Least squares are used for determination of the first set of control points over a set of 3D points obtained from the first pair of images [10], after that a new set of control points is obtained and so on, from one set to the other the 3D-snake is moved. Equation (3) represents such repetition, where the set $V_{t}$ is calculated based on the old set embedded in $F_{\text {ext } 3 D}^{c p}$ as shown in (4).

$$
\begin{align*}
& \mathrm{Q}=\mathrm{NV}  \tag{1}\\
& \mathrm{E}(\mathrm{Q})=\int_{0}^{1} \mathrm{E}_{\mathrm{int}}(\mathrm{Q})+\mathrm{E}_{\mathrm{ext}}(\mathrm{Q}) \mathrm{ds}  \tag{2}\\
& \mathrm{~V}_{\mathrm{t}}=(\mathrm{H}+\gamma \mathrm{I})^{-1}\left(\mathrm{~F}_{\text {ext }}^{\mathrm{cp}}-3 \mathrm{D}\right)  \tag{3}\\
& \mathrm{F}_{\mathrm{ext}-3 \mathrm{D}}^{\mathrm{cp}}=\gamma \mathrm{V}_{\mathrm{t}-1}-g\left(Q_{t-1}\right) \tag{4}
\end{align*}
$$

## B. Obtaining the external force in the space of control points

In (3), $F_{\text {ext-3D }}^{c p}$ corresponds to the vector of 3D external forces in the space of control points and capable to act upon them for guiding the 3D-snake to adjust its projections in accordance to the current pair of images. To obtain such vector firstly is necessary to compose the ordinary vector of 3D external forces ( $F_{\text {ext-3D }}$ ) from the pair of current vector maps and transform these forces to space of control points giving $F_{e x t-3 D}^{c p}$ in demand. This process is described next.

Considering Fig. 1, the point E (obtained by (1)) has the points $E_{1}$ and $E_{2}$ as its projections upon the pair of current vector maps. Assuming that $\mathrm{F}_{\text {ext-2D }}(\mathrm{E} 1)$ is the image force associated to $\mathrm{E}_{1}$ by the vector map number 1 and the same for $\mathrm{F}_{\text {ext-2D }}(\mathrm{E} 2)$ about the vector map number 2, then the points $\mathrm{F}_{1}$ and $\mathrm{F}_{2}$ are homologous points and, this way, the retroprojection [12] of them gives the point F . The vector $E F$ corresponds to the wanted $\mathrm{F}_{\text {ext-3D }}$, this one is transformed to the space of control points by (5) [11] and the vector $V g(Q)$ corresponds one 3D external force that acts upon the control points. The vector $F_{\text {ext }-3 D}^{c p}$ represents the set of all 3D external forces in the space of control points.
$\mathrm{g}(\mathrm{Q}) \cong \mathrm{N}^{\mathrm{T}} \mathrm{F}_{\mathrm{ext}-3 \mathrm{D}}(\mathrm{Q})$

## C. The stiffeness matrix

The matrix H represents the stiffness of the 3D-snake model. The parameters $\alpha$ and $\beta$ which control the flexibility are embedded in H , so H acts on the control points determining the flexibility of the 3D-snake. Cañero [11] suggests calculating H according to (6).

$$
\begin{equation*}
\mathrm{H}=1 / \mathrm{L} \sum_{\sigma}^{\mathrm{L}-1} \mathrm{G}_{\sigma}^{\mathrm{T}}\left(\mathrm{~N}_{\sigma}^{\mathrm{S}}\right)^{\mathrm{T}}\left(\alpha \mathrm{P}^{\prime}+\beta \mathrm{P}^{\prime}\right) \mathrm{N}_{\sigma}^{\mathrm{S}} \mathrm{G}_{\sigma}^{\mathrm{T}} \tag{6}
\end{equation*}
$$

a) The scalar $L$ is the number of spans in the vector of knots of the B-spline;
b) The $N_{\sigma}^{S}$ matrices of span can be calculated algorithmically such as it is described in [13];
c) Matrix $G_{\sigma}$ represents a span matrix with dimensions $d$ times $N B$ ( $d$ is the B-spline's order and $N B$ its number of basis functions) and used for selecting one subset of control points consecutively. The matrix $G_{\sigma}$ is described in (7) and (8). In (8) $\mathrm{m}_{\mathrm{i}}$ corresponds to the multiplicity of the i-th knot in the vector of knots of the B -spline and $d$ is its order.
$\left(\mathrm{G}_{\sigma}\right)_{\mathrm{ij}}=\left\{\begin{array}{ccc}1 & \mathrm{if} & \mathrm{j}-\mathrm{b}_{\sigma}=\mathrm{i} ; \\ 0 & \text { otherwise }\end{array}\right.$
$b_{\sigma}=\left(\sum_{0}^{\sigma} m_{i}\right)-d$
d) The matrix P rises from the development of the calculation of the norm of a B-spline and [11] uses its first and second derivatives, respectivelly matrices $\mathrm{P}^{\prime}$ and $\mathrm{P}^{\prime \prime}$, for determination of internal energy associated to the first and second order of continuity of the B -spline (active contour). The matrices $\mathrm{P}^{\prime}$ and $\mathrm{P}^{\prime \prime}$ are obtained by the (9) and (10).

$$
\begin{align*}
& P^{\prime}=\left\{\begin{array}{cl}
0 \quad \text { if } i=1 & \text { or } \quad j=1 ; \\
\frac{(i-1)(j-1)}{i+j-3} & \text { otherwise }
\end{array}\right.  \tag{9}\\
& P^{\prime \prime}=\left\{\begin{array}{c}
0 \text { if } \mathrm{i}<3 \text { or } \mathrm{j}<3 ; \\
\frac{\left(2-3 \mathrm{i}+\mathrm{i}^{2}\right)\left(2-3 \mathrm{j}+\mathrm{j}^{2}\right)}{\mathrm{i}+\mathrm{j}-5}
\end{array}\right. \text { otherwise } \tag{10}
\end{align*}
$$

## III. VALIDATING THE PROPOSAL

Some initial experiments were done with analytical functions evolving in 3D space [10]. In these cases the true length of each curve was knew and could be compared with the respective measure obtained by 3Dsnake, the points of the 3D-snake were generated by


Figure 1. The image forces $\mathrm{F}_{\text {ext-2D }}$ represented in vector maps are used for recovering the external force $F_{\text {ext-3D }}$ which is converted by $\phi^{-1}$ to the space of control points.
by DeBoor's algorithm [14] and the sum of the Euclidean distance between two adjacent points gave the measure of the length. The measures obtained by 3D-snake approach deviates at most $3 \%$ of the actual ones which was considered acceptable, on the basis of such results, experiments with true electrical arcs were implemented.

Two experiments numbered as 1906 and 1899 were conducted in the facilities of CEPEL, for each experiment, a high current electrical arc was generated and the spatial evolution of it was captured by a pair of cameras.

The experiments with true electrical arcs presented in this paper used a pair simple handycams (Sony ${ }^{\text {T.M. }}$ model HDR-SR10) with low sample rate (approximately 30 fps ) and resolution of $720 \times 570$ pixels (after conversion from manufacturer video format to a popular usable format, such as AVI). The images are manually synchronized after extraction of frames sequence. The experiments were carried out in open environment using actual high voltage transmission towers.

## A. Image processing

In this work the feature of interest in each image is the medial-axes which characterizes the electrical discharge. Taking advantage of the bright of the discharge (Fig. 2), it is applied a threshold for segmenting it from the gray tone original image and the medial-axes result from the morphological thinning and pruning applied to these segmented images. Besides, the distance transform is used for generating a matrix whose


Figure 2. Image of a recent generated electrical arc and its histogram. The brightest pixels corresponds to the arc.
cells represent a pixel in the respective image and store the distance from this pixel to the medial-axes obtained.

Then, this map of distances is operated by the gradient originating the vector map associated to its respective image. Each vector map is responsible for external forces, such as, $\mathrm{F}_{\text {ext-2D }}\left(\mathrm{E}_{1}\right)$ and $\mathrm{F}_{\text {ext-2D }}\left(\mathrm{E}_{2}\right)$ in Fig. 1.

## B. Automatic initialization

The electrical arc is generated along the fuse wire and it resembles a straight line linking two points (the ends of the insulator string). At this initial moment the medial-axes of the discharge can be understood as a 3D vector whose extremities are points $A$ and $B$. Fortunately, this vector AB can be recovered by triangulation of two pairs of homologous points projected at the pair of images taken. This is not a hard problem considering that the projections of the arc look like straight lines too. So that, it is a simple task to detect the extremities of such projections and triangulate them by retroprojection [12] in order to get A and B and the equation of the wanted vector AB . With such equation is generate a set C of points, so that these points constitutes the reconstructed electrical discharge in 3D at its initial instants of existence. The points in the set C are approximated by a third order B -spline $\left(\mathrm{B}_{0}\right)$ the first spatial representation of the 3D-snake.

## C. 3D-snake adjustment and measurement

The 3D-snake evolves itself according to the minimization of its total energy functional described in (3). The snake converges to a stable configuration when a minimum of energy has been obtained which means that the evolution should stop because equilibrium of the internal and external forces has been reached. The goal here is a tracking operation, so the snake needs to find the equilibrium for each pair of vector maps available.

Being a model for the actual media-axes of the discharge, the 3D-snake should track it and the lengths can be estimated by measuring the B -spline contained in the 3D-snake model. The points of the B-spline should be generated by DeBoor's algorithm [14] and the sum of the Euclidean distance between two adjacent points gives the measure of the length.

Because the actual three-dimensional medial-axes representing the electrical discharge is unknown, the accuracy of this tracking must be evaluated by calculating how much each projection of the 3D-snake is adjusted to respective image of the current pair of images. For this sake the B-spline embedded in the 3Dsnake model is projected upon the current pair of maps of distances. Each map provides a vector with the distances between the projection and the respective medial-axes that characterizes the image of a discharge. Then, these pair of vectors is concatenated in one big vector of which norm is calculated providing the measure of the adjustment wanted.

## IV. CALIBRATING THE CAMERAS

The 3D-snake model demands knowledge of the projection matrices of the pair of cameras. Considering the tower and the coordinates of a set of 3D points (marks) determined in its structure, for calibration of each camera, an image depicting at least seven of these points must be captured, after that each projection is
manually identified by inspection upon the image (automatic methods are not efficient in this case because of the latticework complex structure of the tower). The correspondence between each 3D point and its respective projection gives a projection equation system which solution is the wanted projection matrix. Algorithms such as the one described in [15] can be applied for determination of the projection matrix.

## V. Results

There is no way to get the actual length of an electrical arc, however according to [1],[2] and [3] the voltage between arc terminals per unit length is almost constant in a long arc such as the arcs analyzed here. This means that the effective first pseudo harmonic order arc voltage, $\mathrm{V} 1_{\text {rms }}$, can be considered roughly proportional to the arc length, therefore, apart a scale factor, the curve representing the lengths of the arc via 3D reconstruction (3D-snake) has a profile similar to the curve $\mathrm{V} 1_{\mathrm{rms}}$ and this feature can be used for analysis. For the two experiments, both curves are plotted in the same
 in kilovolts. Although it is impossible to compare meters with kilovolts, a similar profile were noticed between both curves, as can be seen, e.g., at Fig. 3 presenting the results of experiment number 1906. The lengths of the discharges measured by 3D-snake method rise from four meters (the length of the insulator string where the arcs started) to four times this initial length which is a consistent behavior, since the wind and the convection heat force the plasma to elongate.


Figure 3. For the experiment 1906, plotted at the same graph: V1rms $[\mathrm{kV}]$ and length [ m ] measured by 3D-snake. The highlighted region at voltage curve represents the disturbance in electrical data at the beginning of the voltage measurement.

## VI. CONCLUSIONS

Since Brazilian national power system presents a natural scenario for SPAR usage, after year 2000 the regulatory agency (ANEEL) imposed the SPAR procedure for every new line project, specific studies became a national concern, a mathematical model of the electrical arc has been pursued since then and the variation of the length of the arc has been considered an important parameter. The present paper describes an innovative application of 3D-snakes that can be used for
tracking, modeling and measuring the length of an electrical discharge.

Although the system acquisition used was far from ideal, the results obtained show the potential of the strategy proposed and demonstrate advantages upon the traditional methods found in literature. New results will be presented soon with faster and synchronized cameras.

## AcKNOWLEDGMENT

The authors thank support received from FURNAS, ANEEL, UDESC, COPPE/UFRJ, CEPEL, FAPESP (The State of São Paulo Research), from CNPq (The National Council for Scientific and Technological Development) and from CAPES (Coordination of Improvement of Higher Level Education Personnel).

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